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Model-Based Algorithms for Detecting Cable Damage from Time-Domain Reflectomertry Measurements

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Time-Domain Reflectometry Measurements
Albuquerque, NM, United States
May 7, 2007 through May 10, 2007

Disclaimer

Model-Based Algorithms for Detecting Cable Damage from Time-Domain Reflectometry Measurements

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NSED, SYSTEMS AND DECISION SCIENCES SECTION

MAY 2, 2007

THIS WORK WAS PERFORMED UNDER THE AUSPICES OF THE U.S. DEPARTMENT OF ENERGY BY THE UNIVERSITY OF CALIFORNIA, LAWRENCE LIVERMORE NATIONAL LABORATORY UNDER CONTRACT NO. W-7405-ENG-48.

Disclaimer and Auspices Statements

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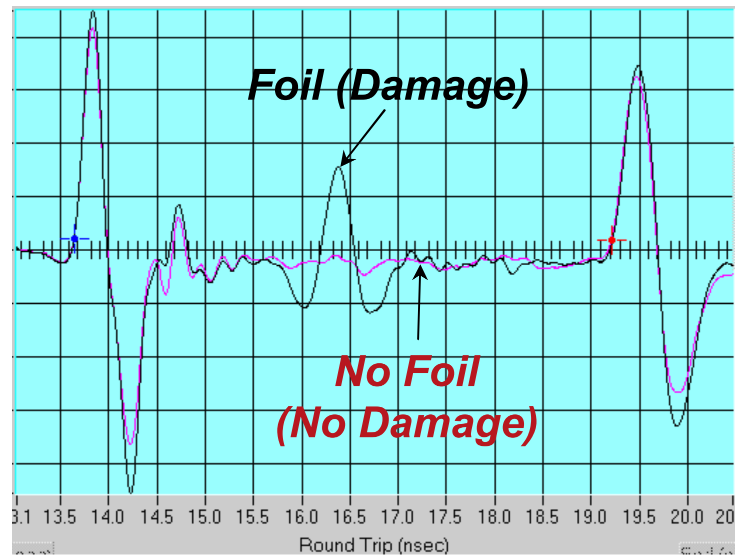
Agenda

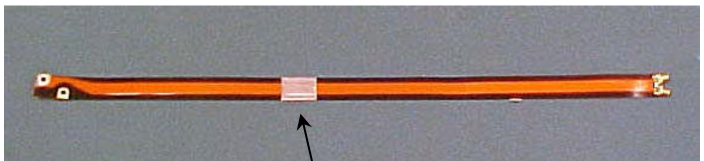
- **Introduction and Problem Definition** ***- Work in Progress***
- **Technical Approach -** ***Model-Based Damage Detection***
- **Model-Based Damage Detection Results**
- **Discussion**

We Are Testing Two-Conductor Flat Cables With Kapton Insulation

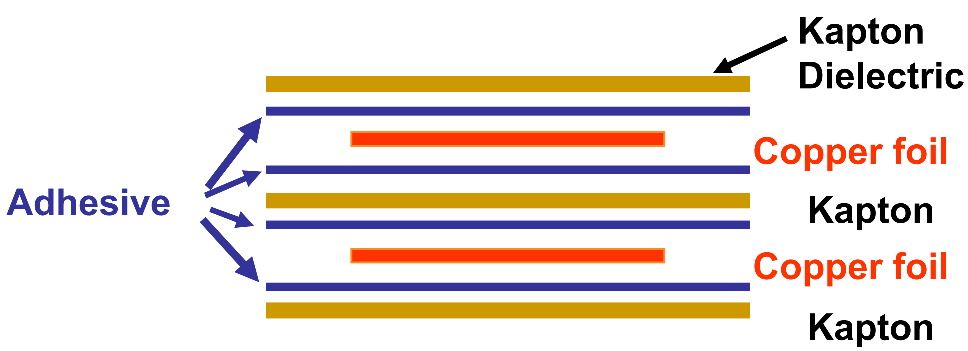
Red TDR Signal => Good Cable
Black TDR Signal => Damaged Cable



Two-Conductor Flat Cable With Kapton Insulation



Foil Simulating a Capacitive Discontinuity (Damage)



Benchtop Experiments (w/No Device “Mockup):”
Connections Create Some Variability

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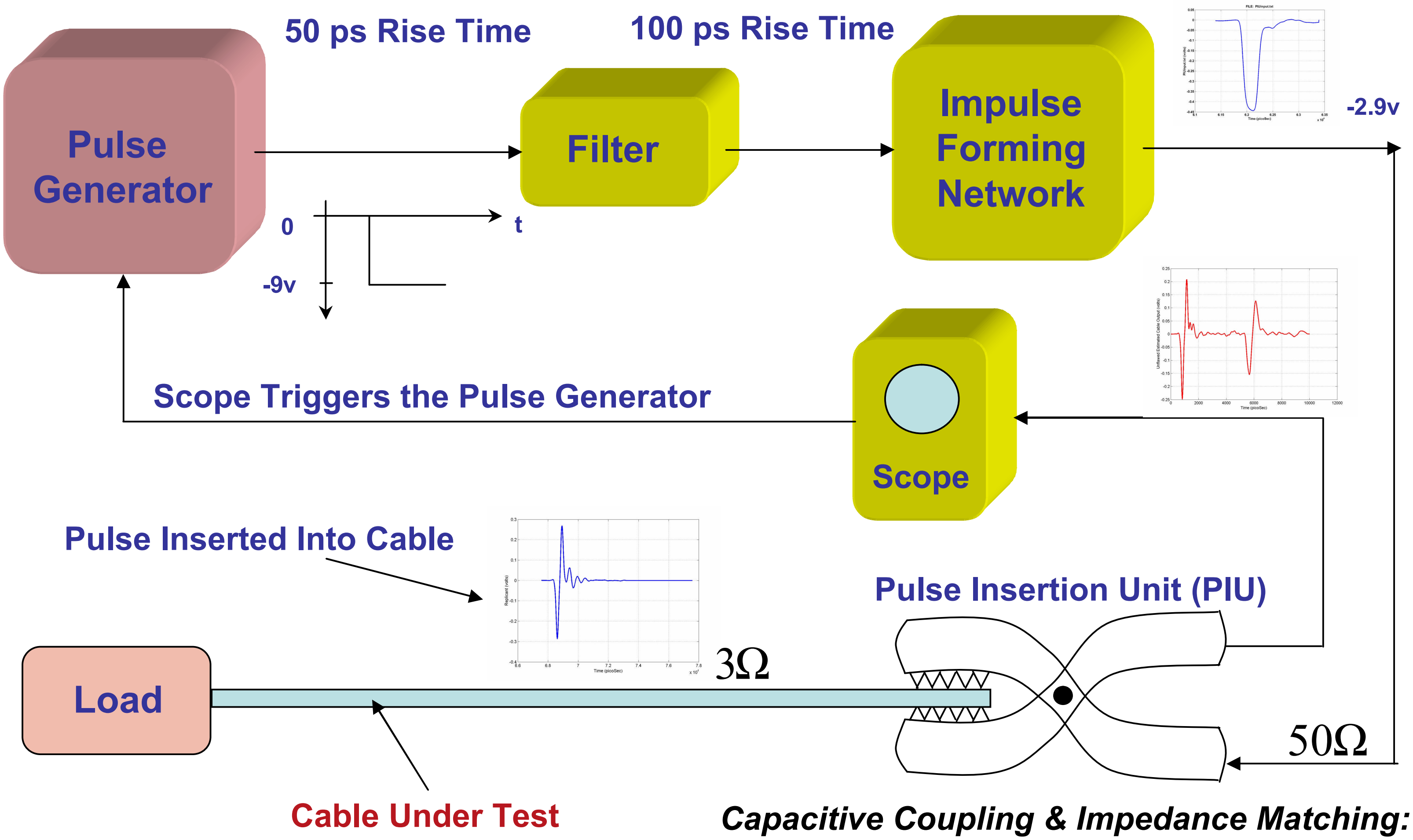

Capacitive Coupling & Impedance Matching:
- ***PIU = Half of “The Capacitor”***
- ***Cable = Half of “The Capacitor”***

Proposed Decision-Making Protocol (Using TDR Measurements):

Use a Three-Step Hierarchical Decision Scheme:

1. ***Detection:***
   - ***Decide whether or not an abnormality in the cable TDR response exists (yes or no)***
   - ***Assume that an abnormal TDR response implies a flaw in the cable***

2. ***Flaw or Failure Mode Classification:***
   - ***Classify the type of failure mode or flaw detected, from among a fixed set of possible modes***

3. ***Final Decision:***
   - ***Using all of the information from the measurements and the previous two steps (fusion), decide whether the cable is "reliable or not reliable"***

Model-Based Damage Detection:
Detect a Model Mismatch if a Damage is Present

- Exploit the fact that the TDR measurements are reasonably repeatable.

- Build a forward model of the dynamic system (cable) for the case in which *NO DAMAGE* exists

- Whiteness Testing on the *Innovations (Errors):*
  Estimate the output of the actual system using measurements from a dynamic test.

  - If *no damage* exists, the model will match the measurements, so the "innovations" (errors) will be *statistically white*.

  - If a *damage* exists, the model will not match the measurements, so the "innovations" (errors) will *not be statistically white.*

- Weighted Sum Square Residuals (WSSR) Test:
  The WSSR provides a single metric for the model mismatch

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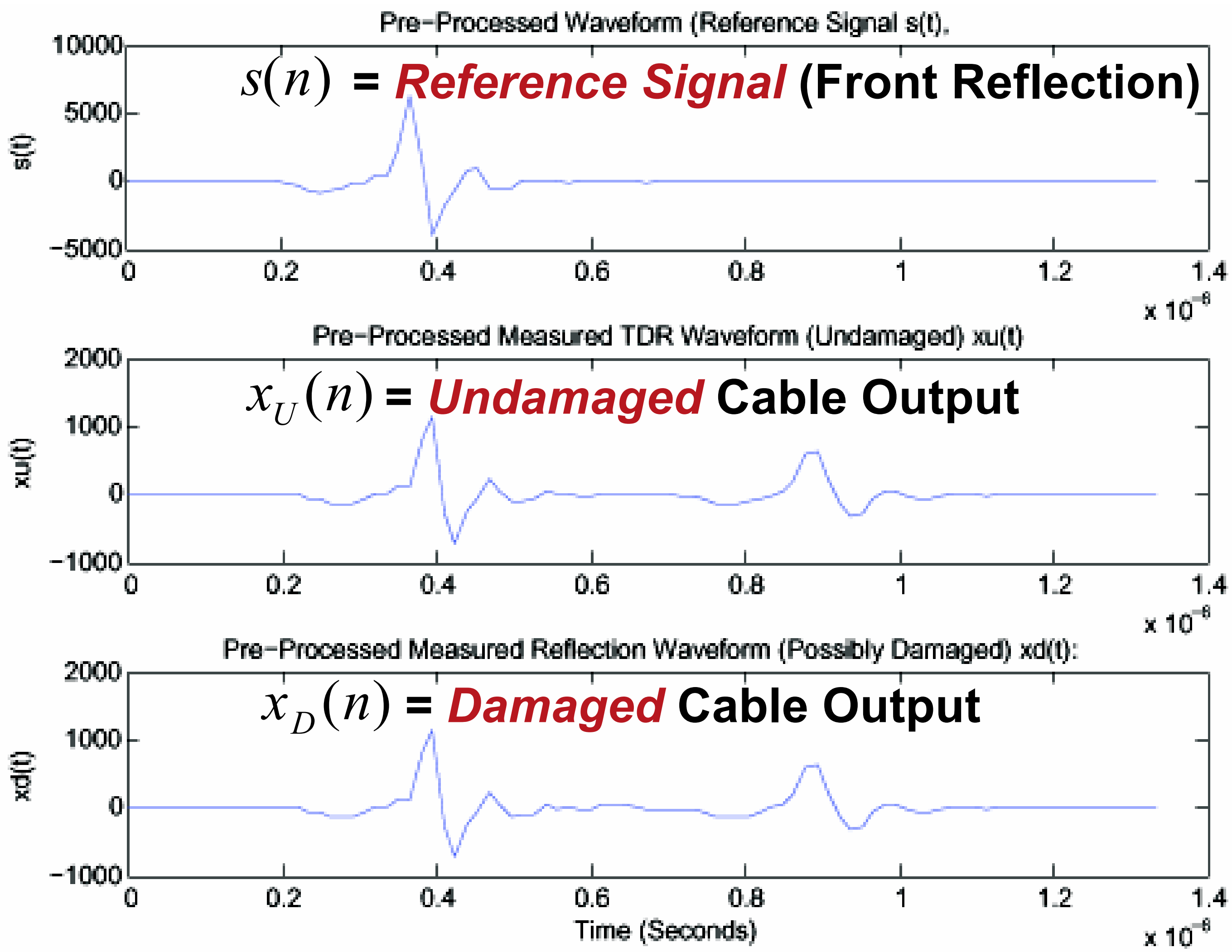
Experiment Using Real Cable TDR Signals:
Pre-Processed Measurements



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Step #1: System Identification to Estimate the Dynamic Model of the *Undamaged Cable*

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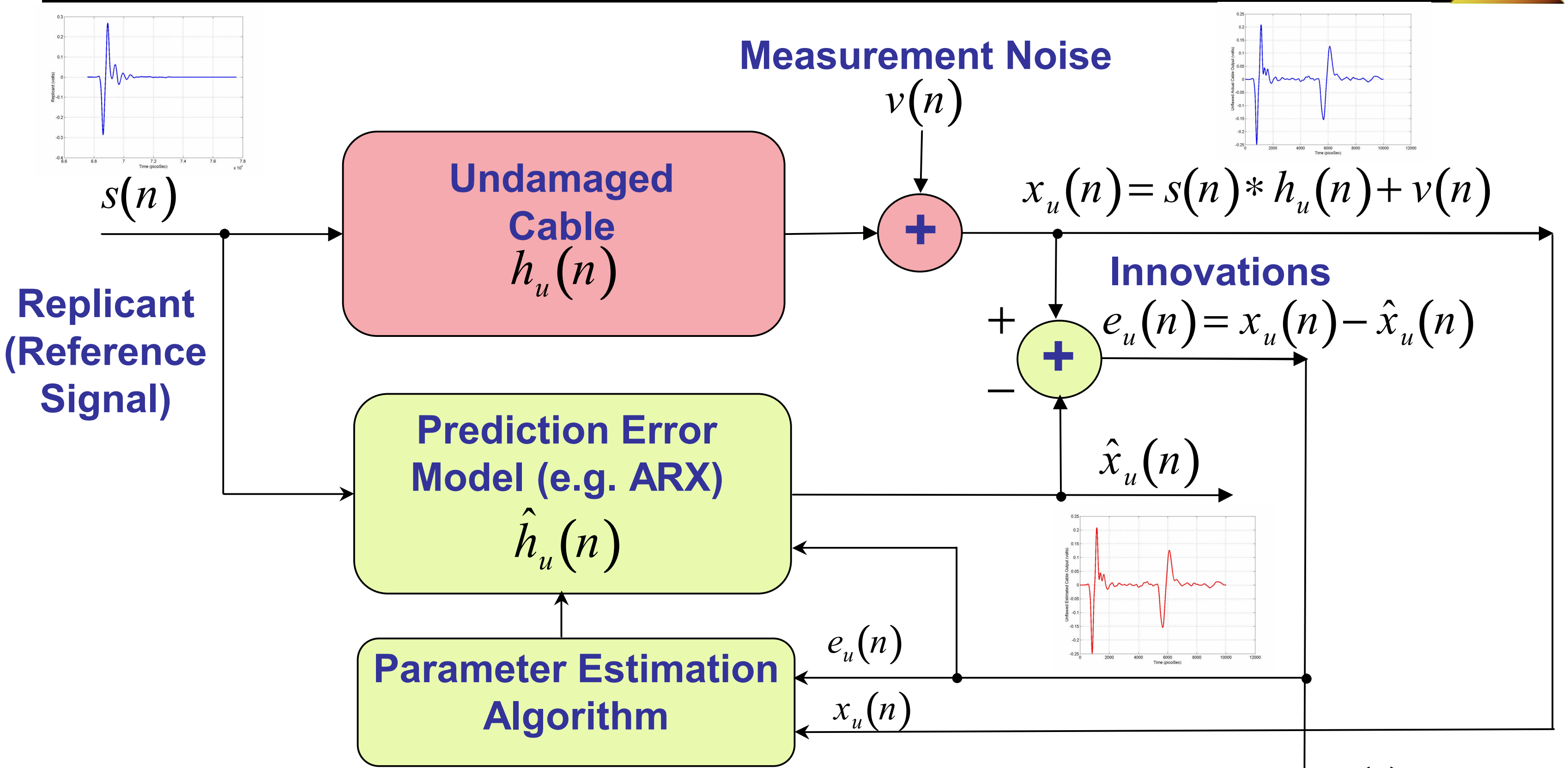
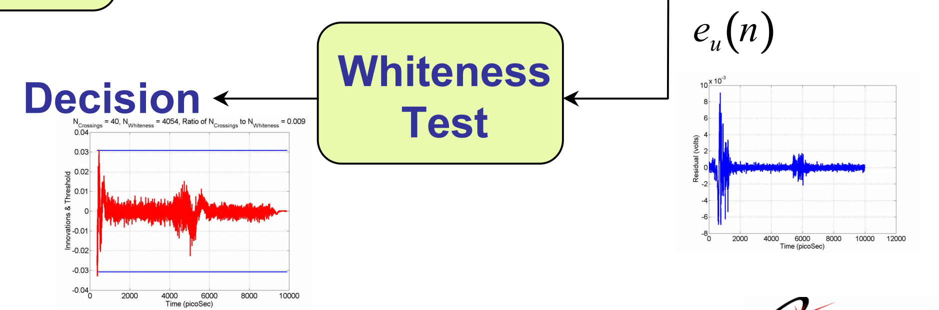
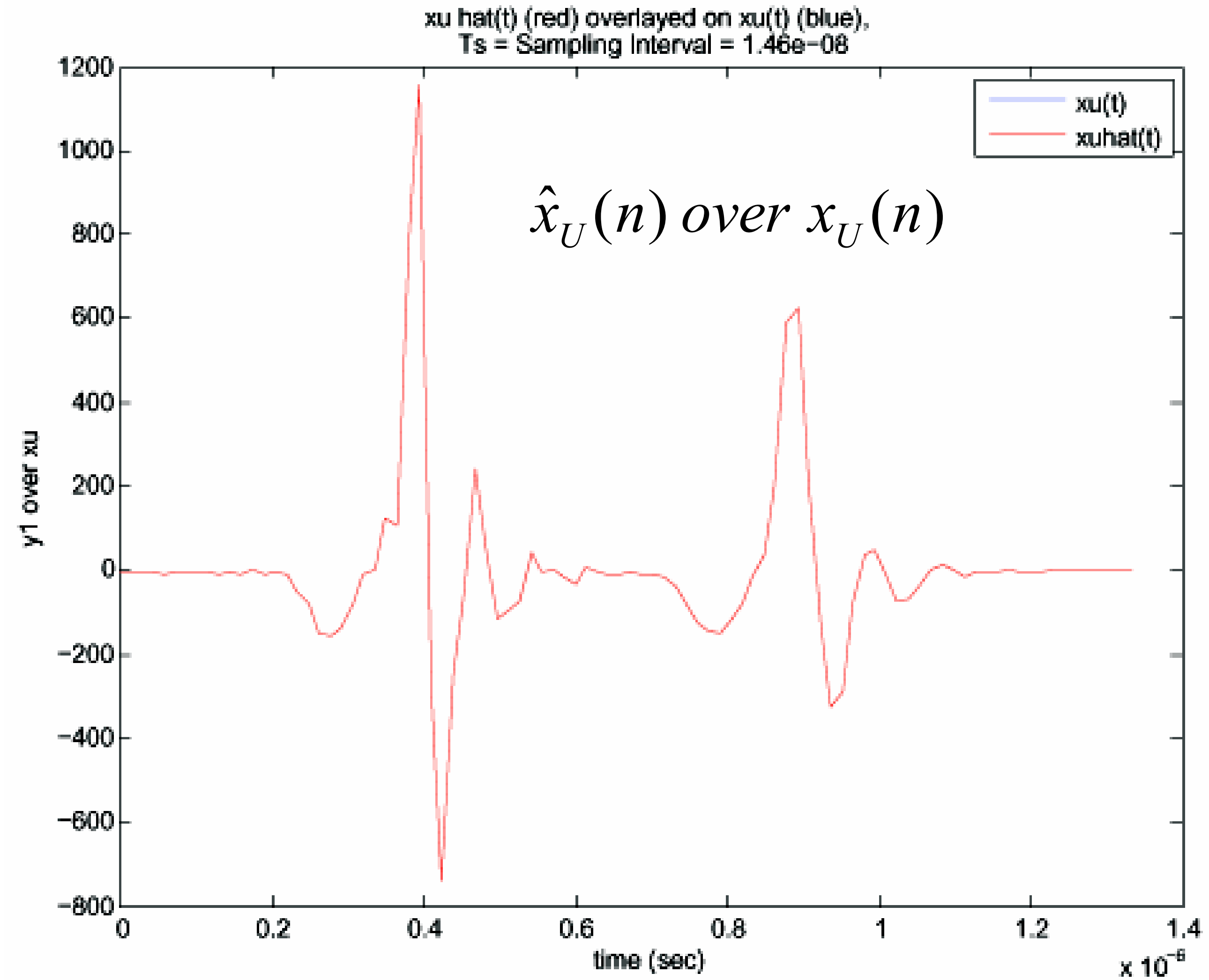




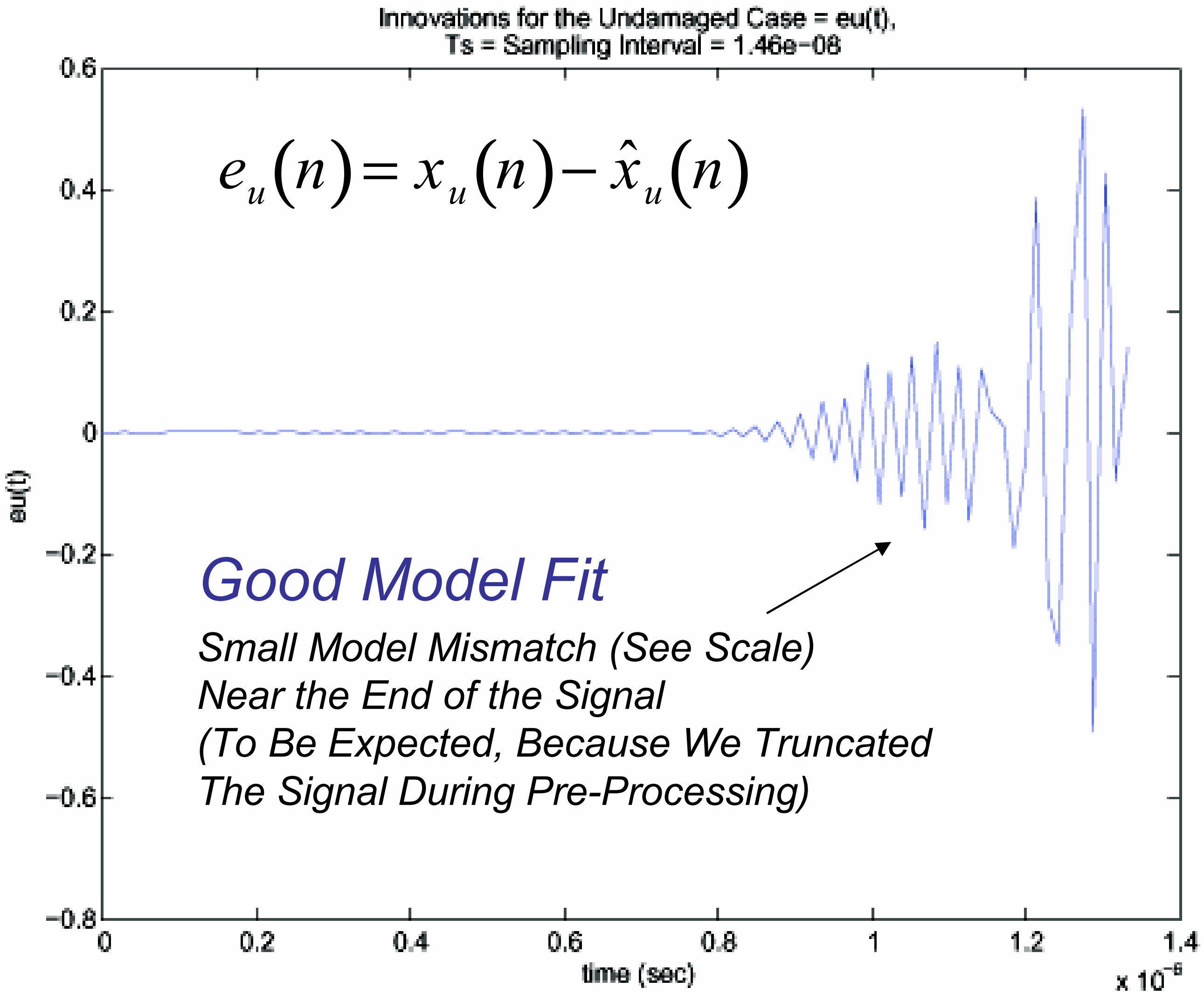
System Identification:

- **Given:** $s(n)$ **and** $x_u(n)$
- **Estimate:** $\hat{h}_u(n)$
- **Test Innovations for whiteness**

Undamaged Case:
$\hat{x}_U(n)$ **Plotted Over** $x_U(n)$ *(Good Model Fit)*



Undamaged Case:
$e_u(n)$ = Residual (or "Innovations")



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Undamaged Case:
Whiteness Test on the Innovations $e_u(n)$

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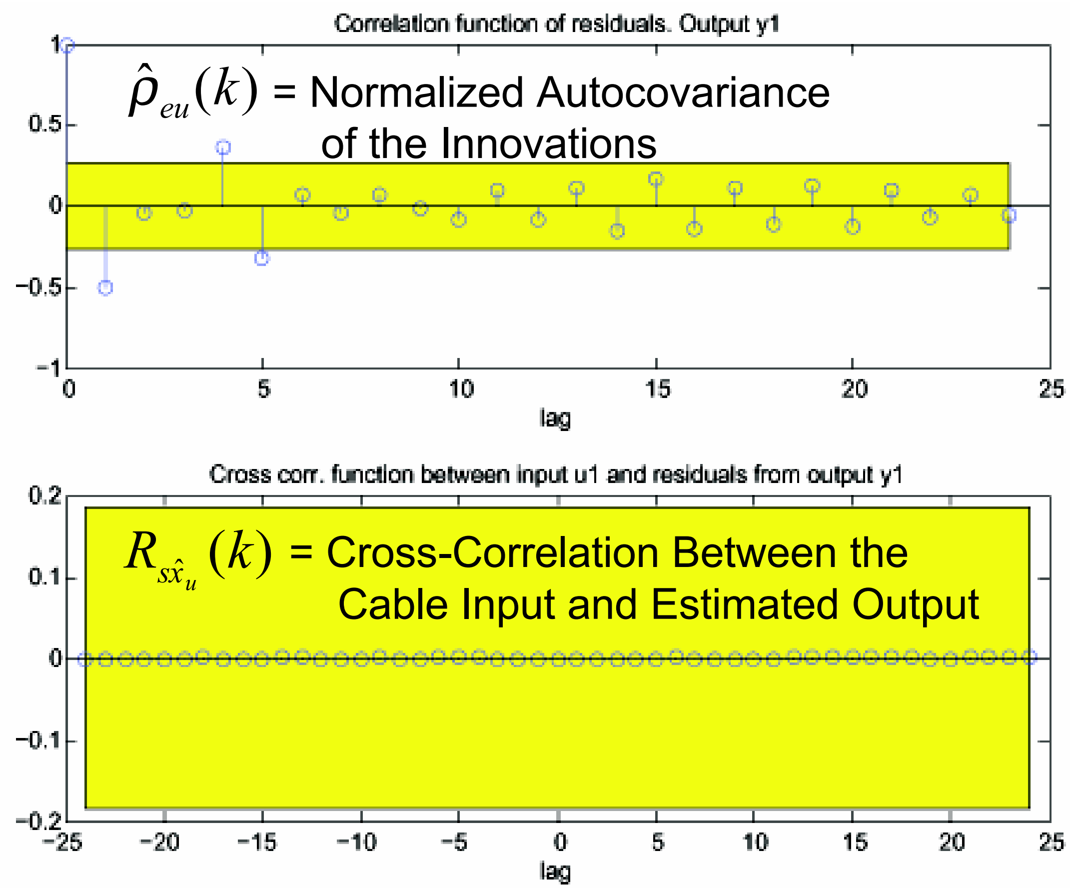
$$e_u(n) = x_u(n) - \hat{x}_u(n)$$ **= *Innovations***



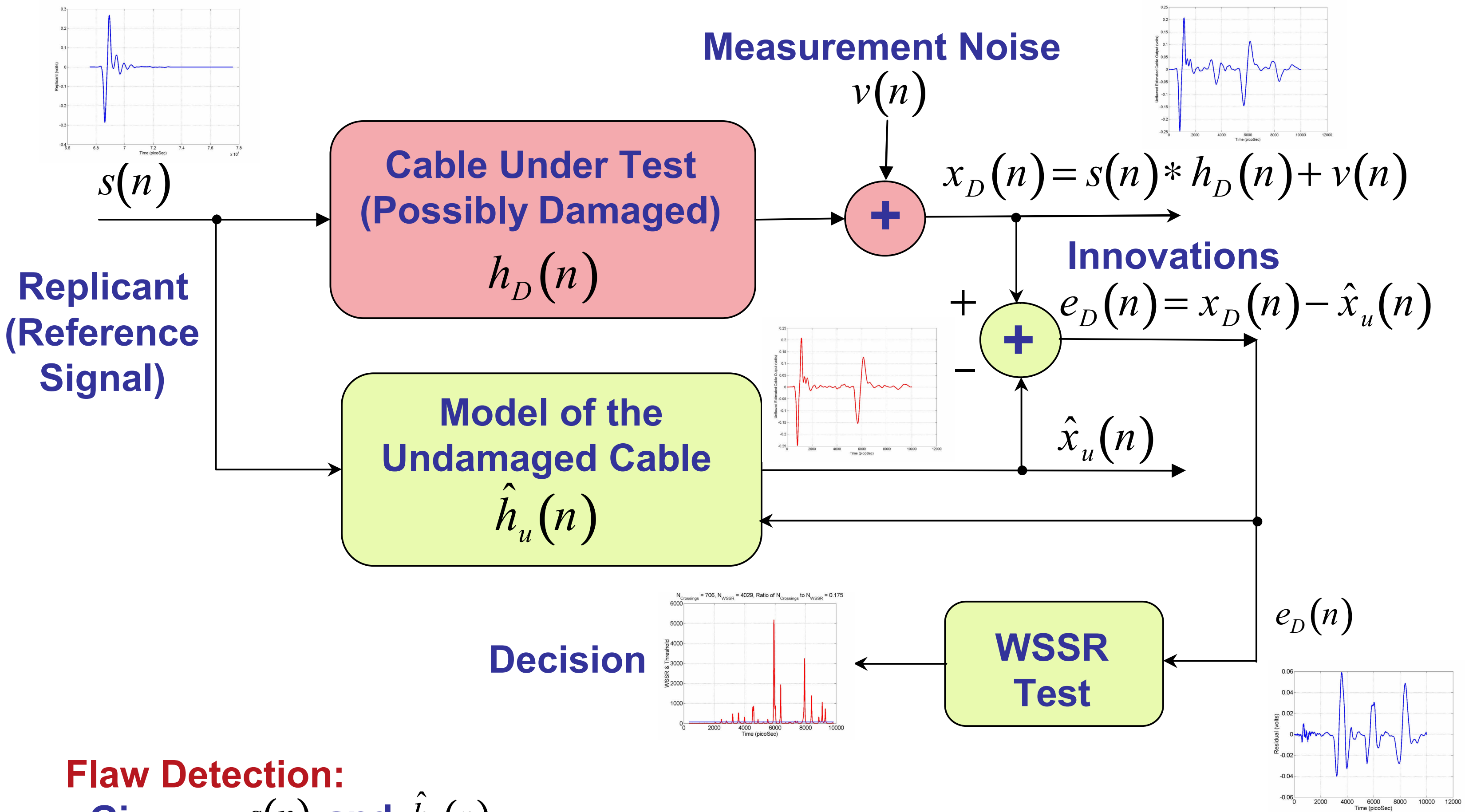
- The innovations pass the Whiteness Test
- The Cross-Correlation is Very Small

$\Longrightarrow$ Declare that the Innovations are "White"

$\Longrightarrow$ There is no model mismatch

$\Longrightarrow$ ***The model is valid***

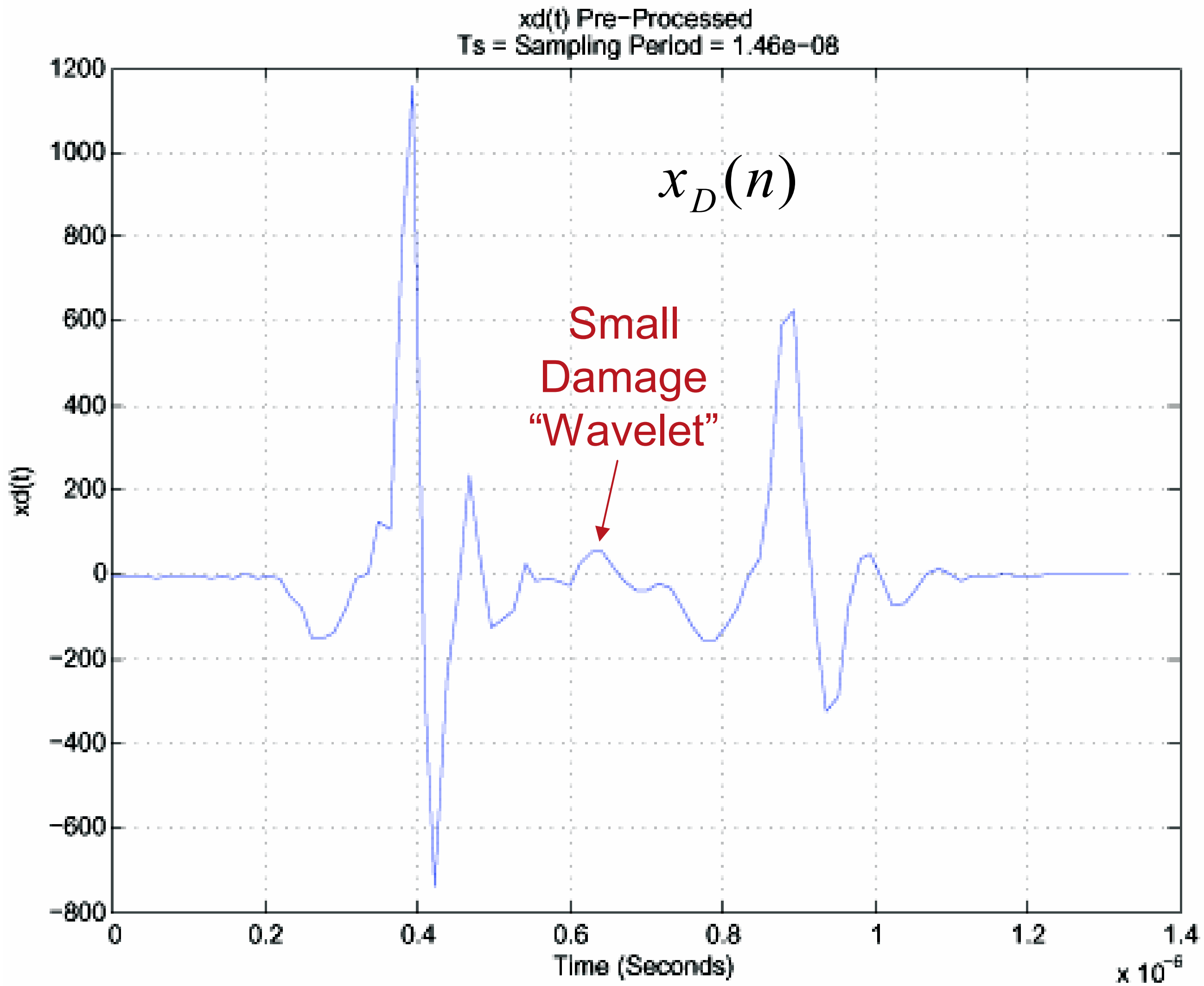
Step #2: Compare the Responses of the Undamaged and Damaged Cables ==> *Damaged Detection*



Flaw Detection:

- **Given:** $s(n)$ **and** $\hat{h}_u(n)$
- **Detect flaws by testing the innovations (nonstationary) for whiteness using WSSR (Weighted Sum Squared Residuals) over a moving window**

Damaged Case:
$x_D(n)$ = *Damaged* Cable Output



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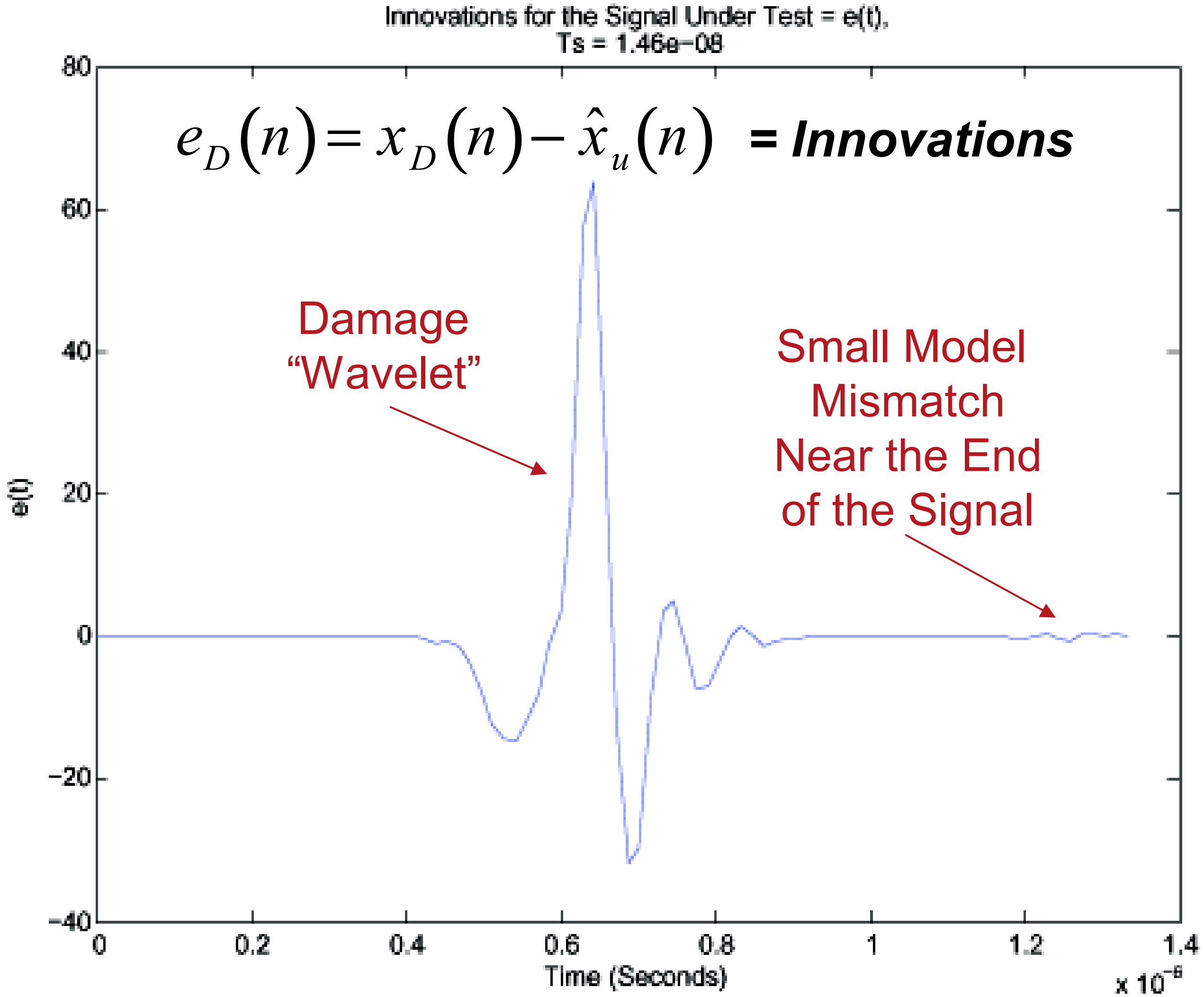
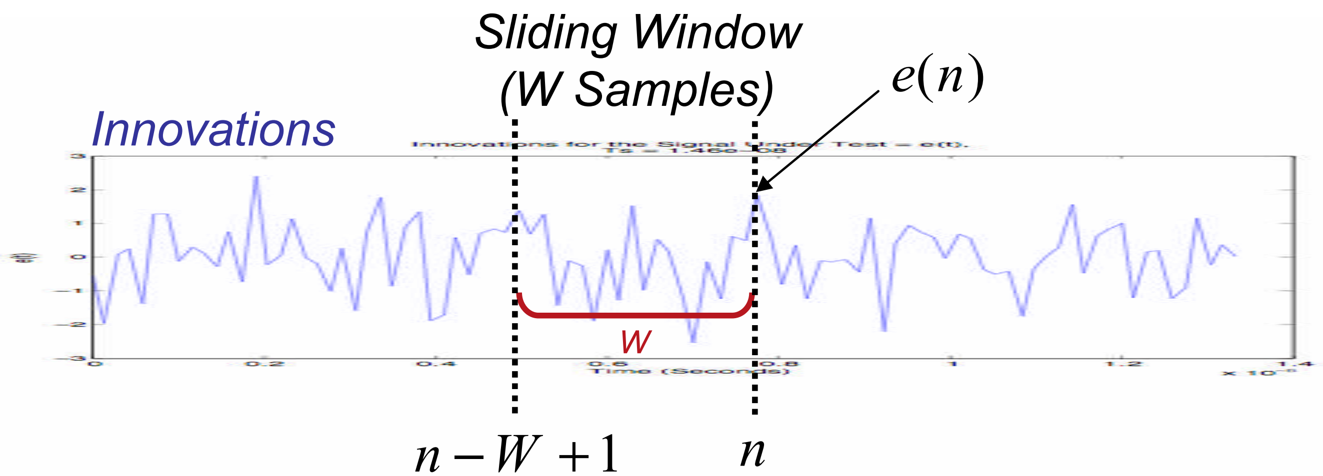
Damaged Case:
$e_D(n)$ = Residual (or "Innovations")



WSSR is Calculated Using a Sliding Window Over the Innovations Sequence $e(n)$

WSSR = "Weighted Sum Squared Residuals"



$$\gamma(n) = \sum_{j=n-W+1}^{n} \frac{e^2(j)}{V(j)}, \quad \text{for } n \geq W$$

WSSR is a useful test statistic for detecting an abrupt change, or "jump" in the innovations

The Scalar WSSR Test *(Continued)*

Summary of the WSSR Test for Significance $\alpha = .05$ **:**

$$\gamma(n) = \sum_{j=n-W+1}^{n} \frac{e^2(j)}{V(j)}, \quad \text{for } n \geq W$$

$$V(n) = \frac{1}{W} \sum_{j=n-W+1}^{n} \left[e^2(j) - \bar{e}(j)\right]^2, \quad \text{for } n \geq W$$

$$\bar{e}(n) = \frac{1}{W} \sum_{j=n-W+1}^{n} e(j), \quad \text{for } n \geq W$$

$$\tau = W + 1.96\sqrt{2W}$$

$$\text{If } \gamma(n) \underset{< H_0}{\overset{\geq H_1}{}} \tau \quad , \quad (\tau = \text{Decision Threshold})$$

In practice, we implement the WSSR test as follows:

- Let F_E = Fraction of samples of $\gamma(n)$ that exceed the threshold
- If $F_E \leq \alpha$, Declare H_0 is true (innovations are white, no jump)
- If $F_E > \alpha$, Declare H_1 is true (innovations are not white, jump)

Damaged Case:
WSSR Test For the Damaged Case

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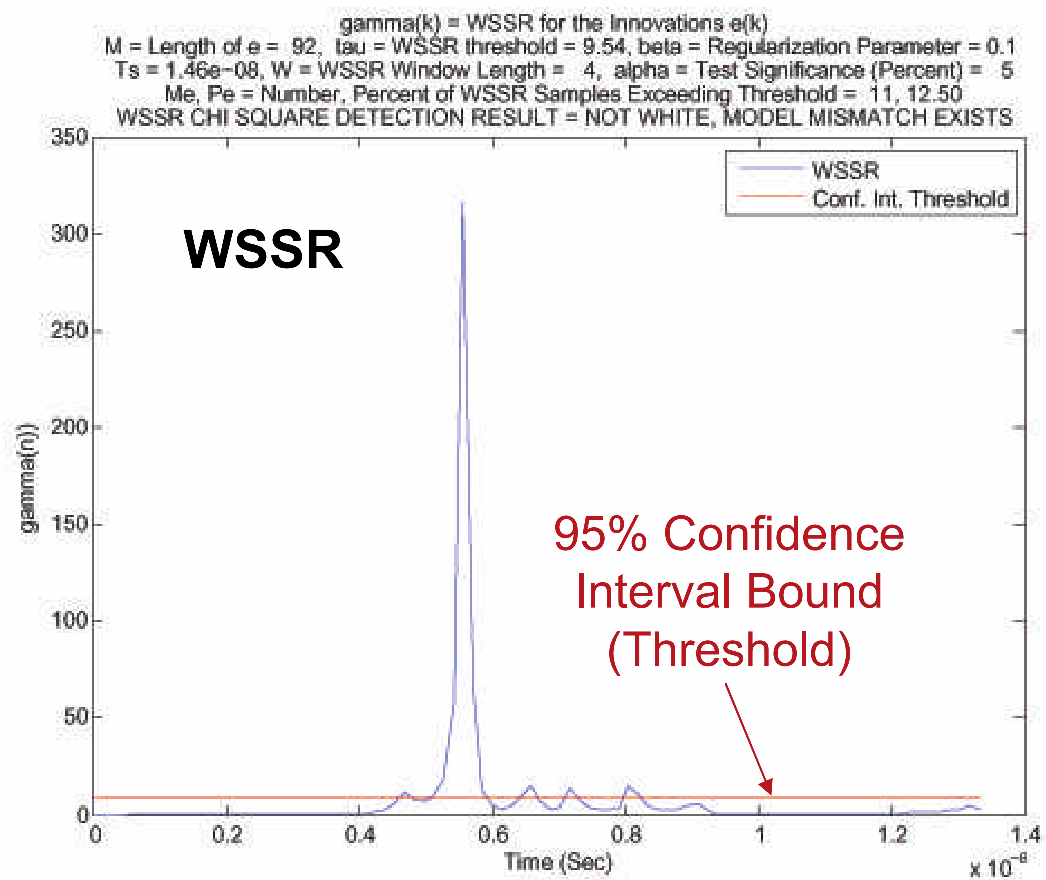
$$\gamma(n) = \sum_{j=n-W+1}^{n} \frac{e^2(j)}{V(j)}, \quad \text{for } n \geq W$$

WSSR = Weighted Sum Squared Residuals



The Innovations Fail the WSSR Test

> 5% of Samples Exceed Threshold

$\Longrightarrow$ There exists a model mismatch

$\Longrightarrow$ *The undamaged model is **NOT** Valid for this cable*

$\Longrightarrow$ ***An anomaly exists in the cable***

Discussion: The Model-Based Approach Offers Advantageous Properties

- We can estimate the *LOCATION* of any detected anomaly.

- The algorithm is *robust* with respect to variations in the measured signals for various experimental scenarios:

  ==> If the TDR signals vary for various scenarios, we can model each case and test the cables effectively.

- This algorithm is very effective, even if we are given *only a single exemplar* of an undamged cable signal.

Discussion: Future Work:

- Thorough repeatability studies:
  - Measurement-to-measurement for one cable
  - Cable-to-cable

- Given ensembles of measurements,
  we can build more extensive performance measures:
  - Receiver Operating Characteristic (ROC) curves
    Probability of Detection
    vs. Probability of False Alarm
  - Statistical Confidence Interval about the estimated
    probability of correct classification

- Experiments in a cable environment (not just bench-top)

- Cable “insult” studies using estimated damage types

Contingency VG’s

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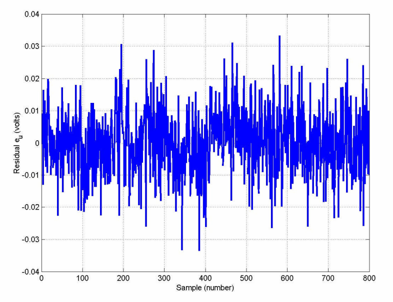
Let Us Define a "White Noise" Sequence $x(t)$

Given a stochastic process $x(t)$



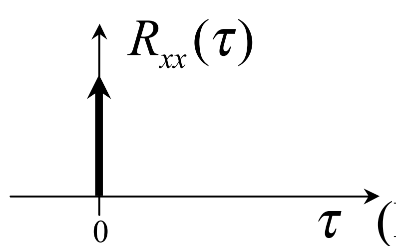
x(t) is "white" when:

Autocorrelation (Time Domain)

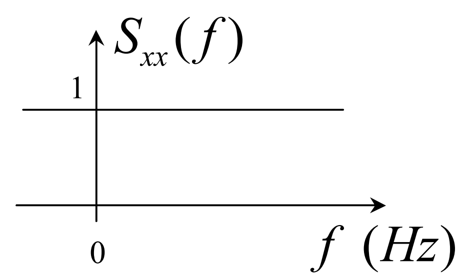
$$R_{xx}(\tau) = E\{x(t)x(t+\tau)\}$$
$$= \delta(\tau)$$
$$= \begin{cases} 1, & \tau = 0 \\ 0, & \tau \neq 0 \end{cases}$$

Power Spectral Density (Frequency Domain)

$$S_{xx}(f) = F\{R_{xx}(\tau)\}$$
$$= 1$$
$$F\{\bullet\} = \text{Fourier Transform}$$



$$\overset{F\{\bullet\}}{\Longleftrightarrow}$$



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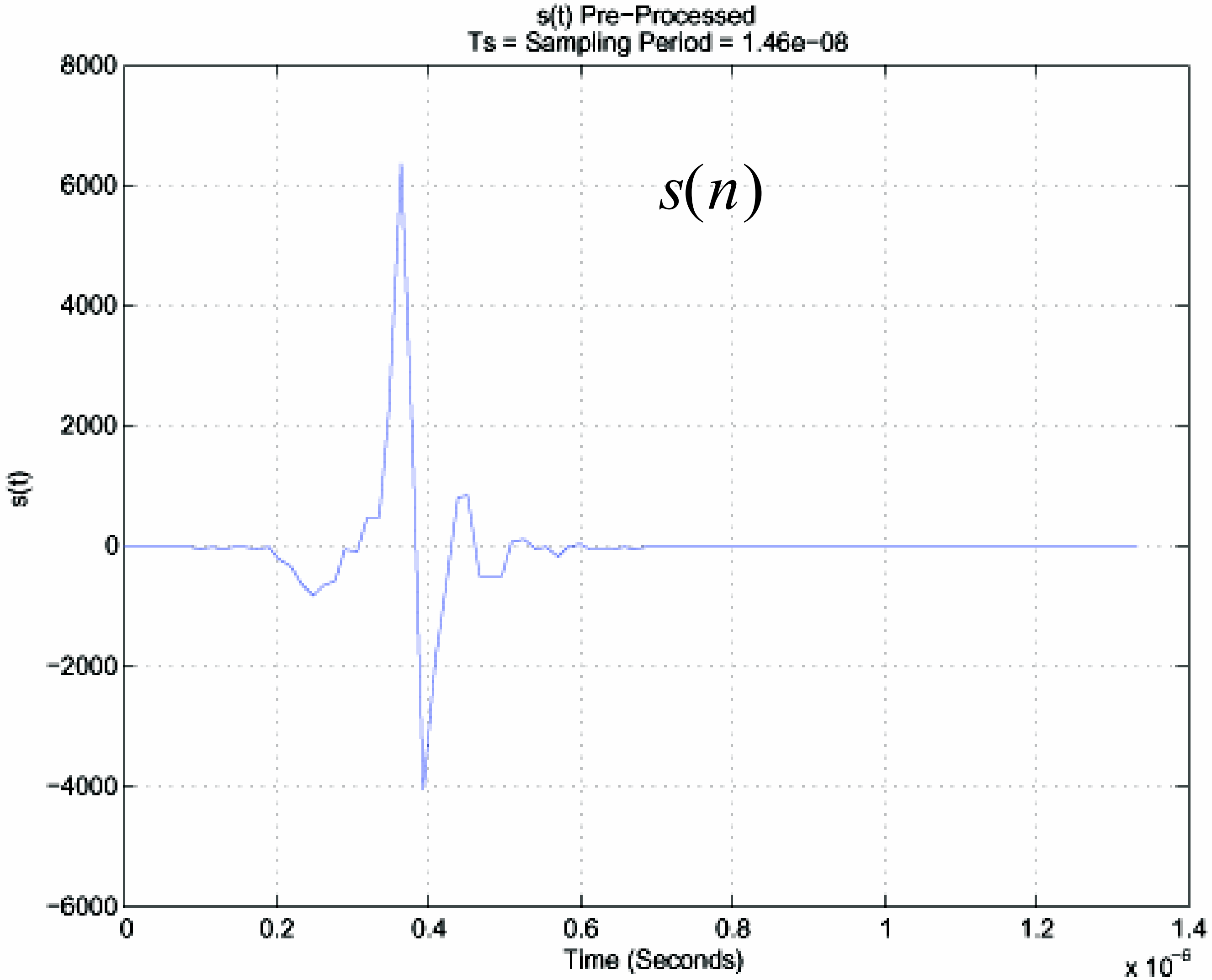
$s(n)$ = ***Reference Signal*** **(Front Reflection)**



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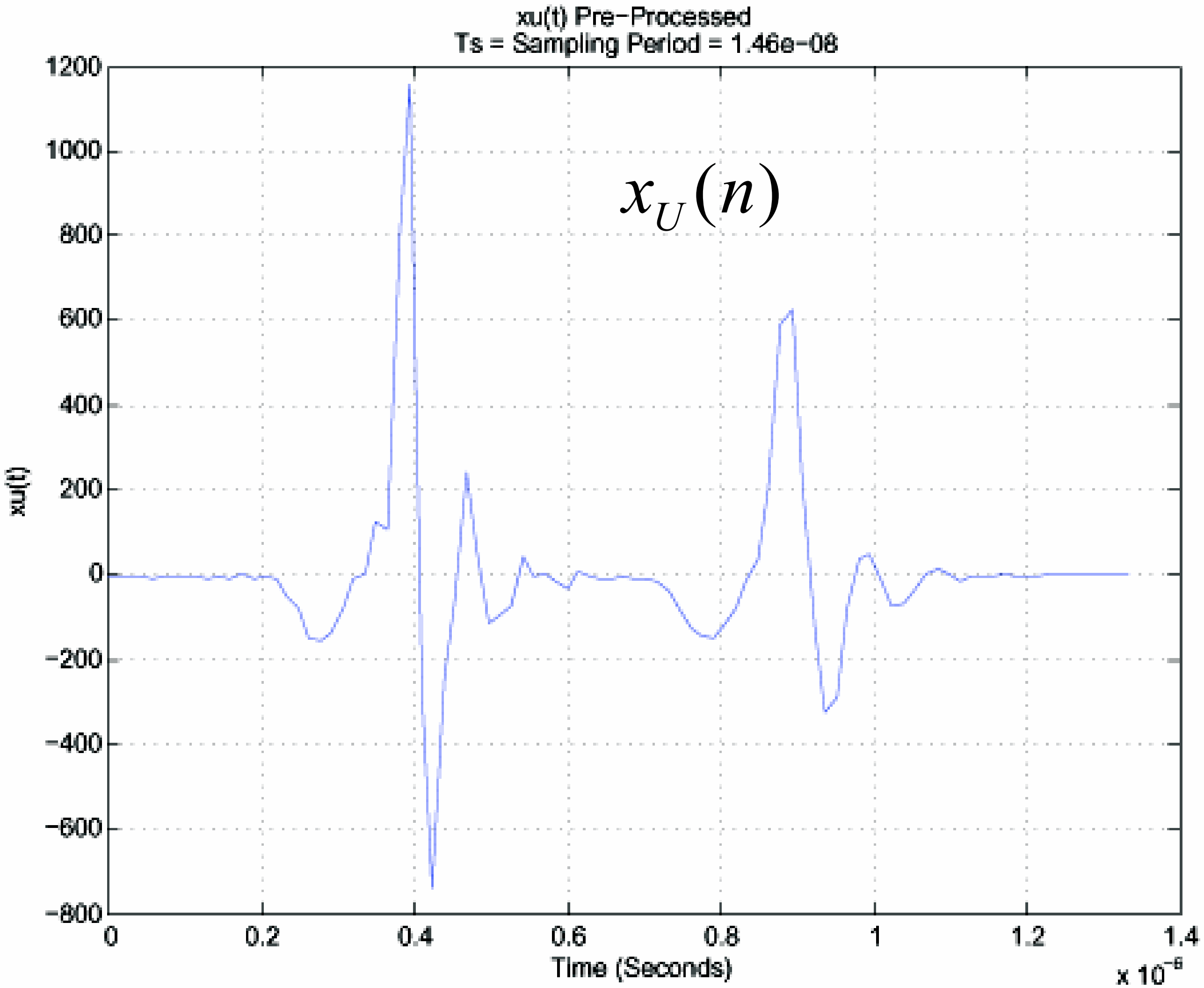
Undamaged Case:
$x_U(n)$ = ***Undamaged*** Cable Output



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Damaged Case:
Whiteness Test For the Damaged Case

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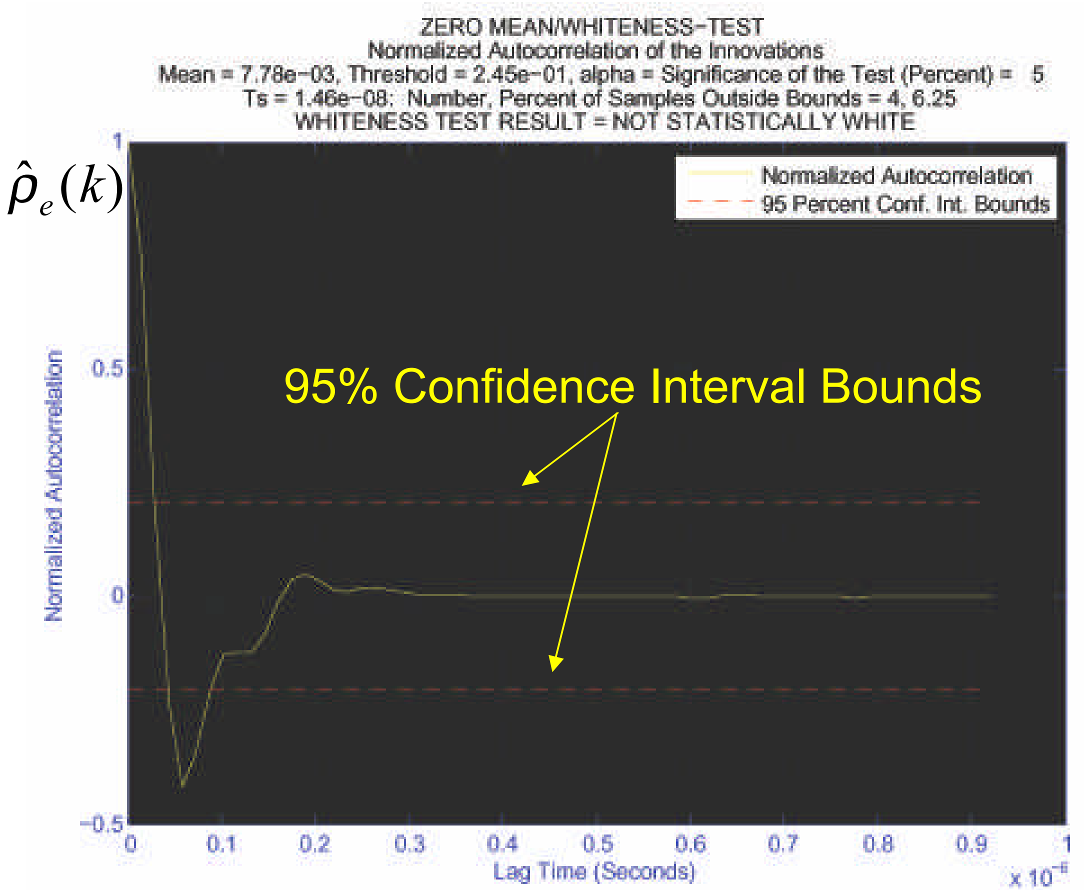
$$e_D(n) = x_D(n) - \hat{x}_u(n)$$ **= *Innovations***



The normalized auto-Covariance $\hat{\rho}_e(k)$ Does Not Pass the 95% Confidence Interval Test

$\Longrightarrow$ Declare that the Innovations are "Not White"

$\Longrightarrow$ There exists a model mismatch

$\Longrightarrow$ *The undamaged model is* ***NOT*** *Valid for this cable*

$\Longrightarrow$ ***An anomaly exists in the cable***

Appendix:
System Identification Using an ARMAX Model

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(Viewgraph in Progress!)

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We Can Write the *Impulse Response h(t)* and *Transfer Function H(z)* of a Linear System

- **Assume that the system is linear and time-invariant**
- **Use the discrete-time system representation**

Input	System	Output
$u(t)$	$h(t)$	$y(t) = u(t) * h(t) = h(t) * u(t)$
$U(z)$	$H(z)$	$Y(z) = U(z)H(z) = H(z)U(z)$

- **The transfer function can be represented by a rational polynomial in the Z-Transform variable, z**

$$H(z) = \frac{B(z)}{A(z)}$$

Where:

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} \cdots + a_{N_a} z^{-N_a}$$

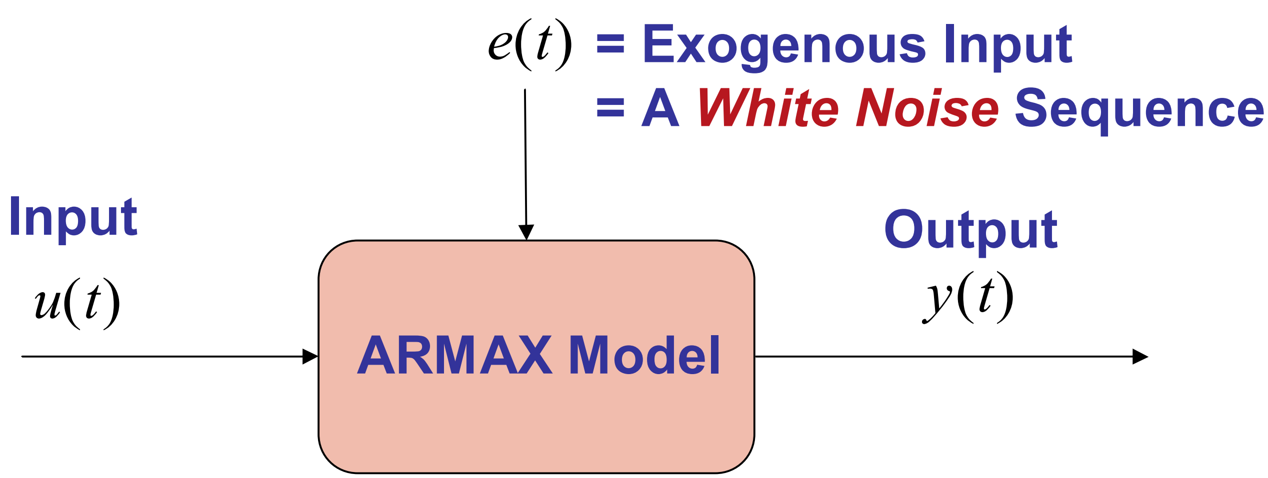
$$B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} \cdots + b_{N_b} z^{-N_b}$$

The General Form of the System Model is Called "ARMAX"

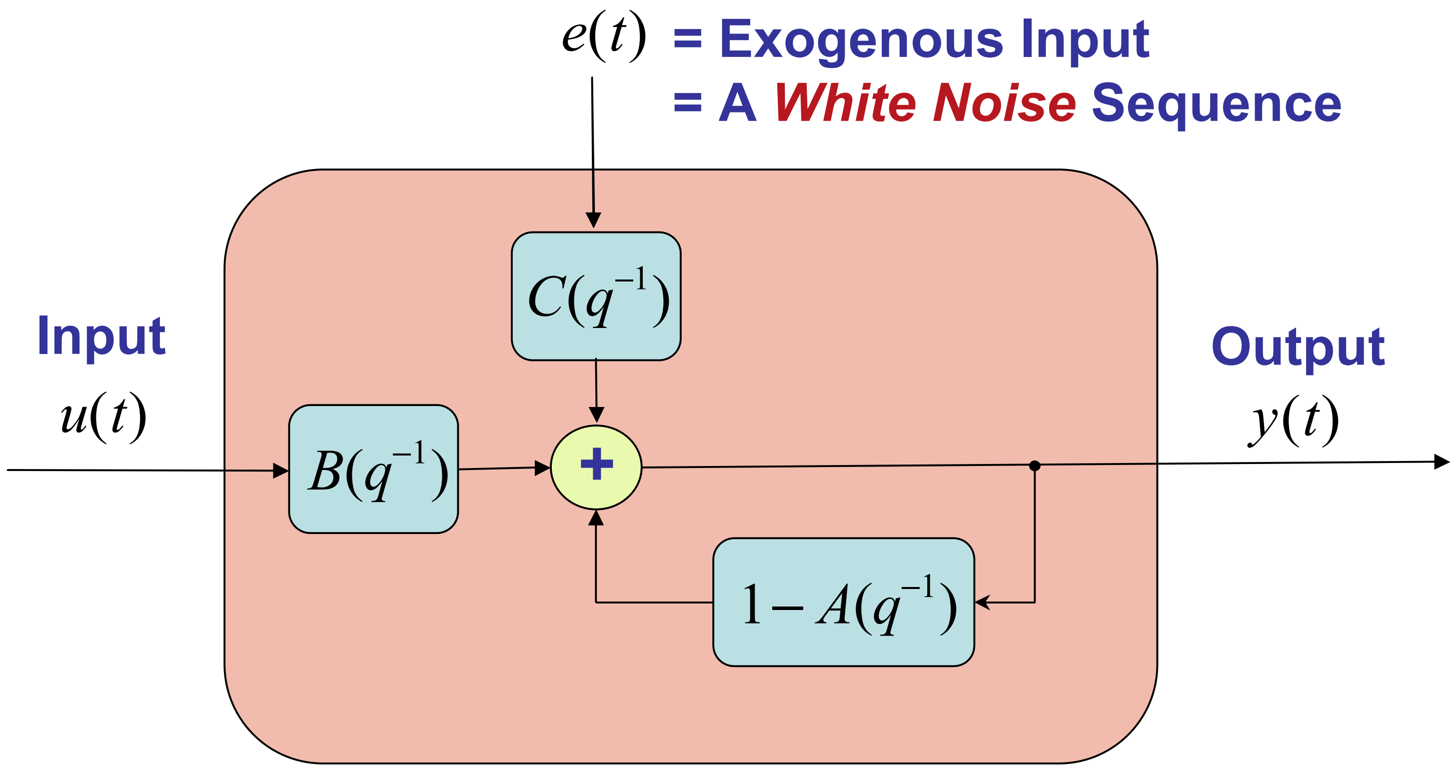
- **ARMAX means "Autoregressive Moving Average with Exogenous Input" ("Exogenous" ==> External)**
- **Let q^{-1} denote the delay operator, so** $q^{-k}y(t) = y(t-k)$
- **The following model is *ARMAX(Na,Nb,Nc):***

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t)$$

Where: $C(z) = 1 + c_1 z^{-1} + c_2 z^{-2} \cdots + c_{N_c} z^{-Nc}$



We Can Draw A Signal Flow Diagram of the ARMAX Model:



WSSR for the Scalar Case

(One Measurement Only, p = 1)

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WSSR (Weighted Sum Squared Residuals) Test

For a Scalar Measurement ($p = 1$)

WSSR is used to deal with two main issues:

1) Multiple measurements ($p > 1$):

WSSR allows us to aggregate multiple whiteness tests into a single aggregated scalar test over all of the measurements.

2) Nonstationary Prediction Errors (Innovations):

When the signals are nonstationary, WSSR is a more reliable statistic to use for testing the whiteness of the prediction error sequence (innovations). We require that the WSSR lies beneath a calculated threshold to deem the innovations zero-mean and white.

WSSR is calculated over a sliding window of W samples. It is a useful test statistic for detecting an abrupt change in the innovations signal

Under the zero mean assumption, the WSSR statistic is equivalent to testing that the prediction error sequence is white.

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Scalar WSSR (Weighted Sum Squared Residuals) Test

For a Scalar Measurement (p = 1)

Given the innovations signal $e(n)$

We define the scalar WSSR test statistic at time index n:

$$\gamma(n) = \sum_{j=n-W+1}^{n} \frac{e^2(j)}{V(j)}, \quad \text{for } n \geq W$$

Note: We estimate WSSR over a finite sliding window of length W samples.

Where:

$$V(n) = \frac{1}{W} \sum_{j=n-W+1}^{n} \left[e^2(j) - \bar{e}(j) \right]^2, \quad \text{for } n \geq W$$

Sample variance over the sliding window

$$\bar{e}(n) = \frac{1}{W} \sum_{j=n-W+1}^{n} e(j), \quad \text{for } n \geq W$$

Sample mean over the sliding window

Define the WSSR Hypothesis Test

By defining a threshold (later), the WSSR test becomes:

$$\text{If } \; \gamma(n) \underset{< H_0}{\overset{\geq H_1}{}} \tau \quad , \qquad (\tau = \text{Decision Threshold})$$

Read this as follows:

$$\begin{aligned} &\text{If } \; \gamma(n) \geq \tau, \;\; \text{then } H_1 \;\; \text{is true} \\ &\text{If } \; \gamma(n) < \tau, \;\; \text{then } H_0 \;\; \text{is true} \end{aligned}$$

WSSR Test

For a scalar measurement (p = 1) *(Continued)*

For the null hypothesis H_0, the WSSR is chi square distributed:

$$\gamma(n) \sim \chi^2(W)$$

However, for W > 30, the WSSR is approximately normally distributed:

$$\gamma(n) \sim N(W, 2W)$$

At the significance level α , the probability of rejecting the null Hypothesis (detecting a jump) is:

$$P\left(\left|\frac{\gamma(n) - W}{\sqrt{2W}}\right| > \left|\frac{\tau - W}{\sqrt{2W}}\right|\right) = \alpha$$

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WSSR Hypothesis Test (Continued)

At the significance level α, we can create a confidence interval test:

For H_0: $$P\left[\gamma(n) < \tau\right] = 1 - \alpha = .95$$

For H_1: $$P\left[\gamma(n) \geq \tau\right] = \alpha = .05$$

For a significance level $\alpha = .05$, the threshold is:

$$\tau = W + 1.96\sqrt{2W}$$